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Broadcasting with Selective Reduction

and BSR solutions to Geometric Problems

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Outline

1 What is Broadcasting with Selective Reduction
- Introduction to the BSR Model
- Additional Features of BSR
- Comparison of BSR to PRAM

2 BSR Algorithms
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The most powerful model we have examined

If a problem of size n can be solved in $t(n)$ time units using p processors in either an interconnection network or a combinational circuit, that problem can be executed in PRAM in *at most* $t(n)$ using *at most* $p(n)$ processors.

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The PRAM Model consists of:

- N Processors
- M Shared-Memory Units
- Memory Access Unit (MAU)

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Also consists of N Processors, M Memory Units, and an MAU.

- MAU can be built with same width, depth, and size as PRAM
- Memory access and Computation time:
  - Memory access: $\tau_a(N, M) = O(\log M)$ as in PRAM, assumed $O(1)$
  - Computation: $\tau_c(N, M) = O(1)$ as with all models
- Executes any combination of CR, all forms of CW, as well as EW, & EW
- BSR is *at least* as powerful as PRAM

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The BROADCAST Instruction

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In addition to the instructions we are familiar with, **BSR** has an additional instruction called **BROADCAST**, which executes in three phases.

Broadcasting: Each processor P_i broadcasts a $datum, d_i$ and a tag, g_i to all Memory locations. This is an **All-to-ALL** operation from Processors to Memory.

Selection: Each memory location U_j uses a limit, $l_j, 1 \leq j \leq M$, and selection rule σ to test the condition $g_i \sigma l_j$. If $g_i \sigma l_j$ is true, the associated d_i is selected by U_j.

Reduction: All d_i selected by U_j are combined into a single data using a *reduction operator* $\mathcal{R}$.

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notation

$$\underset{1 \le j \le M}{U_j} \quad \longleftarrow \quad \underset{\substack{g_i \ \sigma \ l_j \\ 1 \le i \le N}}{\mathcal{R}} \ d_i$$

Notation for BROADCAST instruction

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Two points of Note

1. **BROADCAST** is equivalent to M CW instructions.
   In BSR, the **BROADCAST** instruction takes $\tau(N, M)$, or $O(1)$, time. On PRAM, these operations would take $M \times \tau(N, M)$
2. **BROADCAST** makes BSR *strictly more powerful* than PRAM.
   We have seen that BSR is at least as powerful, because it can execute all PRAM operations in the same time and requires the same resources. The additional instruction creates a more powerful model.

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BSR is More Powerful the PRAM

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An Example Problem

$X = x_1, x_2, \ldots, x_n,\ \ x_i \geq x_{i+1}$
$L = l_1, l_2, \ldots, l_n,\ \ l_i < l_{i+1}$
To Solve: for $1 \leq i \leq n$, the sum s_i such that $X \neq l_i$

$O(n)$ solution on the RAM

1. Compute the Sum S of all X
2. Merge X, L into Y such that every l_i precedes every element $X = l_i$
3. Scan Y and compute s_i by subtracting all $X = l_i$ from S

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BSR is More Powerful than the PRAM

PRAM Solution

There is no constant time solution to this problem on the PRAM using n processors. n processors can calculate the value of a given s_i, in constant time, but not all of them.

BSR Solution

Broadcast: Processor P_i broadcasts (x_i, a_i) as the (tag, datum) pair.

Select: Memory location U_j selects $x_i \neq l_j$, $1 \leq j \leq n$

Reduce: Using $\sum$ as the operator, each U_j calculates s_j

The BSR solution takes $O(1)$, and does not require that X or L be sorted.

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BSR Prefix Sums

Algorithm BSR Prefix Sum

Using n processors and n memory locations, $\leq$ for selection, $\sum$ as the reduction operator, and j as the limit.

for $j = 1$ to n **do**

  for $i = 1$ to n **do**

BSR Prefix Sums

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The algorithm consists of a single **BROADCAST** instruction. $p(n) = n, t(n) = O(1)$. Cost is therefore $O(n)$, which is optimal.

Example

This would be an excellent opportunity to use the whiteboard for the case $n = 3$

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BSR Sort

Sorting by enumeration

This algorith for BSR sort uses the same approach as the sequential algorithm *counting sort*, that we have seen used to sort with a mesh-of-trees. The problem is to sort an unordered array of numbers $X = x_1, x_2, \ldots, x_n$ into a sequence $S = s_1, s_2, \ldots, s_n, s_i \leq s_{i+1}$

1. In the first step, the rank of each item is calculated. At the termination of this step, each memory location U_j holds the rank, $r_j$for each item.
2. In the second step, x_j is placed in $1 + r_j$ of S. The difficulty is to resolve ties between equal values. A second broadcast using $\bigcap$ resolves this difficulty.

BSR Sort

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Sorting by enumeration

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Algorithm BSR Sort

BSR Sort

for $j = 1$ to n **do**
  $r_j \leftarrow 0$
  for $i = 1$ to n **do**

for $j = 1$ to n **do**
  $r_j \leftarrow r_j + 1$
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BSR Sort

for $j = 1$ to n **do**
    $r_j \leftarrow 0$
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        $r_j \leftarrow \sum_{x_i < x_j} 1$
for $j = 1$ to n **do**
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BSR Sort Analysis

Workout

Figure 11.4, 11.5

Cost Analysis

- Two broadcast steps: cost is $p(n) \times t(n) = O(n)$
- Doesn't this violate the $O(n \log n)$ lower bound on sort?
- Not if we use *discriminating analysis*

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Discriminating Analysis

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- Recall that memory access ($\tau_a(N, M)$) requires $O(\log M)$, where M is the number of memory locations.
- We have n memory locations, and there are a constant number of operations.

$$t(n) = O(1) \times (\tau_a(N, M) + \tau_c(N, M)) = O(\log n)$$

With n processors:

$$c(n) = n \times O(\log n = O(n \log n)$$

which is optimal.

- For the PRAM model, discriminating analysis reveals a cost of $O(n \log^2 n)$, which is not optimal.

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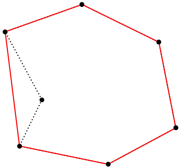
Description of the problem

The convex hull problem is a problem in planar geometry. Given a set of points, find the subset of points that describes a boundary.



CONVEX HULL

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The algorithm takes 10 Broadcast steps. It depends on implementing a multiple criteria BSR, which is admissable. In the first step, each point is examined to determine if it is certainly not on the convex hull. Lines are drawn between the remaining points, with two lines forming the Maximal Angle at the point. Those points with a maximal angle that is convex are on the convex hull.

Our example set

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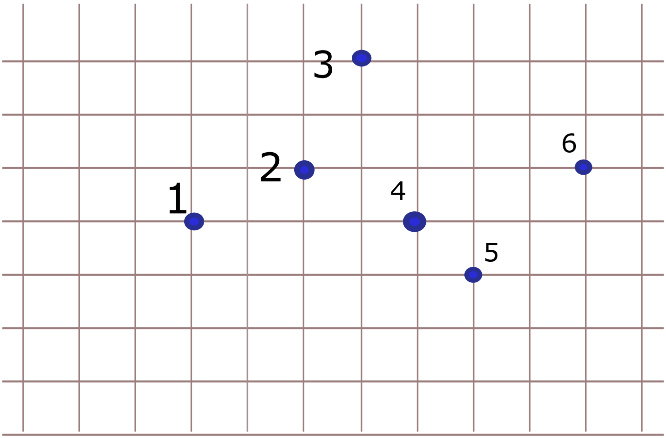
(1, 4) (3, 5) (4, 7) (5, 4) (6, 3) (7, 5)



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Given a set S of N' points on a plane:

Step 1

A_i is equal to 0 if there is no point to the bottom-left of S_i, B_i represents the top left, C & D the right quadrants.

for all $0 < i, i' \leq N'$ **do**

$A_i \leftarrow \bigcap_{X_i \geq X_{i'} \wedge Y_i \geq Y_{i'} \wedge i \neq i'} 1$ $\quad B_i \leftarrow \bigcap_{X_i \geq X_{i'} \wedge Y_i \geq Y_{i'} \wedge i \neq i'} 1$

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	1	2	3	4	5	6
A	0	1	1	1	0	1
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C	1	0	1	1	0	0
D	1	1	0	1	1	0

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Step 2

E_i is equal to 1 if the i^{th} point of S is not in the convex hull.

for $i = 0$ to N' **do**
$E_i \leftarrow A_i \wedge B_i \wedge C_i \wedge D_i$

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Algorithm BSR Convex Hull

Step 3

This step is similar to step 1, but here inequalities are strict.

for all $0 < i, i' \leq N'$ **do**
  if $E_i = 0$ **then**

  else

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for all $0 < i, i' \leq N'$ **do**

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	1	2	3	4	5	6
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C’	1	1	1	1	0	0
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Step 4

$F_{i,j}$ is the reverse of the slope of the straight line containing P_i and P_j

for all $i, j \mid j \leq N'$ **do**

if $(X_i = X_j)$ **then**

else

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for all $i, j \mid j \leq N'$ **do**

  if $(X_i = X_j)$ **then**

    if $(Y_i > Y_j)$ **then**

    else

  else

    if $Y_i = Y_j$ **then**

    else

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Step 4

$F_{i,j}$ is the reverse of the slope of the straight line containing P_i and P_j

for all $i, j \mid j \leq N'$ **do**
  if $(X_i = X_j)$ **then**
    if $(Y_i > Y_j)$ **then**
      $F_{i,j} \leftarrow +\infty$
    else
      $F_{i,j} \leftarrow -\infty$
  else
    if $Y_i = Y_j$ **then**

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  else
    if $Y_i = Y_j$ **then**
      $F_{i,j} \leftarrow 0$
    else
      $F_{i,j} \leftarrow (X_i - X_j)/(Y_i - Y_j)$

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Step 5

G_i and H_i are made up by the reverses of the slopes of the two lines containing P_i, which shape the maximal angle at P_i.

for all $i, j \mid i > 0, j \geq N'$ **do**

if $(E_i = 0)$ **then**

else

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    if $((B_i' = 0 \wedge C_i' = 0) \vee (A_i' = 1 \wedge D_i' = 1))$ **then**

    if $((A_i' = 0 \wedge D_i' = 0) \vee (B_i' = 1 \wedge C_i' = 1))$ **then**

    if $(A_i' = C_i' \wedge B_i' = D_i' \wedge A_i' \neq B_i')$ **then**

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  if $(E_i = 0)$ **then**
    if $((B_i' = 0 \wedge C_i' = 0) \vee (A_i' = 1 \wedge D_i' = 1))$ **then**
      $G_i \leftarrow \bigcap_{F_{i,j}>0} F_{i,j}$ $H_i \leftarrow \bigcup_{F_{i,j}>0} F_{i,j}$
    if $((A_i' = 0 \wedge D_i' = 0) \vee (B_i' = 1 \wedge C_i' = 1))$ **then**
      $G_i \leftarrow \bigcup_{F_{i,j}<0} F_{i,j}$ $H_i \leftarrow \bigcap_{F_{i,j}<0} F_{i,j}$
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      $G_i \leftarrow \bigcup_{F_{i,j}<0} F_{i,j}$ $H_i \leftarrow \bigcap_{F_{i,j}>0} F_{i,j}$
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    if $((B_i' = 0 \wedge C_i' = 0) \vee (A_i' = 1 \wedge D_i' = 1))$ **then**

$$G_i \leftarrow \bigcap_{F_{i,j}>0} F_{i,j} \qquad H_i \leftarrow \bigcup_{F_{i,j}>0} F_{i,j}$$

    if $((A_i' = 0 \wedge D_i' = 0) \vee (B_i' = 1 \wedge C_i' = 1))$ **then**

$$G_i \leftarrow \bigcup_{F_{i,j}<0} F_{i,j} \qquad H_i \leftarrow \bigcap_{F_{i,j}<0} F_{i,j}$$

    if $(A_i' = C_i' \wedge B_i' = D_i' \wedge A_i' \neq B_i')$ **then**

$$G_i \leftarrow \bigcup_{F_{i,j}>0} F_{i,j} \qquad H_i \leftarrow \bigcap_{F_{i,j}>0} F_{i,j}$$

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for all $i, j \mid i > 0, j \geq N'$ **do**

  if $(E_i = 0)$ **then**

    if $((B'_i = 0 \land C'_i = 0) \lor (A'_i = 1 \land D'_i = 1))$ **then**

$$G_i \leftarrow \bigcap_{F_{i,j}>0} F_{i,j} \qquad H_i \leftarrow \bigcup_{F_{i,j}>0} F_{i,j}$$

    if $((A'_i = 0 \land D'_i = 0) \lor (B'_i = 1 \land C'_i = 1))$ **then**

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  if $(E_i = 0)$ **then**
    if $((B'_i = 0 \wedge C'_i = 0) \vee (A'_i = 1 \wedge D'_i = 1))$ **then**
      $G_i \leftarrow \bigcap_{F_{i,j} > 0} F_{i,j} \qquad H_i \leftarrow \bigcup_{F_{i,j} > 0} F_{i,j}$
    if $((A'_i = 0 \wedge D'_i = 0) \vee (B'_i = 1 \wedge C'_i = 1))$ **then**
      $G_i \leftarrow \bigcup_{F_{i,j} < 0} F_{i,j} \qquad H_i \leftarrow \bigcap_{F_{i,j} < 0} F_{i,j}$
    if $(A'_i = C'_i \wedge B'_i = D'_i \wedge A'_i \neq B'_i)$ **then**
      $G_i \leftarrow \bigcup_{F_{i,j} > 0} F_{i,j} \qquad H_i \leftarrow \bigcap_{F_{i,j} > 0} F_{i,j}$
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      $G_i \leftarrow \bigcap_{F_{i,j}>0} F_{i,j}$ $H_i \leftarrow \bigcup_{F_{i,j}>0} F_{i,j}$

    if $((A_i' = 0 \wedge D_i' = 0) \vee (B_i' = 1 \wedge C_i' = 1))$ **then**

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      $G_i \leftarrow \bigcup_{F_{i,j}>0} F_{i,j}$ $H_i \leftarrow \bigcap_{F_{i,j}>0} F_{i,j}$

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      $G_i \leftarrow \bigcap_{F_{i,j} > 0} F_{i,j} \qquad H_i \leftarrow \bigcup_{F_{i,j} > 0} F_{i,j}$

    if $((A_i' = 0 \wedge D_i' = 0) \vee (B_i' = 1 \wedge C_i' = 1))$ **then**

      $G_i \leftarrow \bigcup_{F_{i,j} < 0} F_{i,j} \qquad H_i \leftarrow \bigcap_{F_{i,j} < 0} F_{i,j}$

    if $(A_i' = C_i' \wedge B_i' = D_i' \wedge A_i' \neq B_i')$ **then**

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      $G_i \leftarrow \bigcup_{F_{i,j}<0} F_{i,j}$ $H_i \leftarrow \bigcap_{F_{i,j}<0} F_{i,j}$
    if $(A'_i = C'_i \wedge B'_i = D'_i \wedge A'_i \neq B'_i)$ **then**
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    if $(A'_i = B'_i \wedge C'_i = D'_i \wedge A'_i \neq C'_i)$ **then**
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	1	2	3	4	5	6
G	-5	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{3}{2}$
H	2	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{2}$	6

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Step 6

$-\infty$ is changed into $+\infty$ for a best homogeneity of the values of G and H

for $i > 0$ to N' **do**
  if $(G_i = -\infty)$ **then**

  if $(H_i = -\infty)$ **then**

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for $i > 0$ to N' **do**

if $(G_i = -\infty)$ **then**

$G_i \leftarrow +\infty$

if $(H_i = -\infty)$ **then**

$H_i \leftarrow +\infty$

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Step 6

$-\infty$ is changed into $+\infty$ for a best homogeneity of the values of G and H

for $i > 0$ to N' **do**
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Step 7

K_i is equal to 1 if the value of G_i exists in H. L_i is equal to 1 if the value of H_i exists in G.

for all $i > 0, i' \geq N'$ **do**

  if $(E_i = 1)$ **then**

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Step 7

K_i is equal to 1 if the value of G_i exists in H. L_i is equal to 1 if the value of H_i exists in G.

for all $i > 0, i' \geq N'$ **do**

if $(E_i = 1)$ **then**

$$K_i \leftarrow \bigcap_{\substack{G_i = H_{i'} \\ \wedge i \neq i'}} 1$$

$$L_i \leftarrow \bigcap_{\substack{H_i = G_{i'} \\ \wedge i \neq i'}} 1$$

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	1	2	3	4	5	6
G	-5	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{3}{2}$
H	2	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{2}$	6
K	0	1	1	x	1	0
L	0	1	0	x	0	0

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Step 8

M_i is equal to 1 if the point P_i is in the convex hull

for all $0 < i \leq N'$ **do**

$M_i \leftarrow K_i \wedge L_i$

Step 9

N_i contains indices which will be used in the next step.

for all $i > 0, i' \leq N'$ **do**

$$N_i \leftarrow \sum_{i' \leq i} M_{i'}$$

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K	0	1	1	x	1	0
L	0	1	0	x	0	0
M	1	0	1	0	1	1
N	1	1	2	2	3	4

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Step 10

Xch and Ych contain the coordinates of the points in the convex hull

for all $i > 0, i' \le N'$ **do**

$$Xch_i \leftarrow \bigcap_{\substack{M_{i'}=1 \\ \& N_{i'}=i}} X_{i'}$$

$$Ych_i \leftarrow \bigcap_{\substack{M_{i'}=1 \\ \& N_{i'}=i}} Y_{i'}$$

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K	0	1	1	x	1	0
L	0	1	0	x	0	0
M	1	0	1	0	1	1
N	1	1	2	2	3	4
Xch_i	1	4	6	7	0	0
Ych_i	4	7	3	5	0	0

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